

Effective Lagrangian with vector mesons : Linear response theory

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Abstract

The soliton breathing mode is investigated in the framework of linear response theory within a Skyrme model vector meson stabilized. The effective Lagrangian considered includes the ρ (introduced following the standard prescription of nonlinear chiral symmetry) and the ω mesons. The monopole response function is found to have a pronounced peak which is identified to the $P11$ (Roper) resonance. The results are compared to those obtained within the local approximation.

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In a recent paper [1] we have considered an extended Skyrme model which includes fourth and sixth-order terms and have investigated the soliton breathing mode in the framework of the linear response theory. We have found that the monopole response function exhibits an unbound sharp peak which we have identified to the Roper resonance $N(1440)$. Since the fourth and sixth-order terms can be derived from a local approximation of an effective model with ρ [2] and ω mesons [3, 4] respectively, it seems therefore interesting to explore the soliton breathing mode, with the same method as in [1] (first proposed in Ref. [5]), within an effective Lagrangian which incorporates these two mesons *explicitly*. This is the purpose of the present paper.

1. Our starting point is the following Lagrangian density:

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \mathcal{L}_{\pi\rho} + \mathcal{L}_{\pi\omega} + \frac{1}{4} f_\pi^2 m_\pi^2 \text{Tr}(U + U^\dagger - 2) \quad (1)$$

where U is an $SU(2)$ matrix parametrized by the pion fields π_a , normalized to the pion decay constant f_π :

$$U = \exp[i \vec{\pi} \cdot \vec{\tau} / f_\pi] , \quad (2)$$

the τ_a 's being the usual Pauli matrices. The first term in Eq. (1) corresponds to the well known nonlinear σ model. The second one corresponds to the coupling between the isospin one ρ -meson and the pion field. The ρ vector meson is frequently introduced in the chiral Lagrangian as hidden gauge particle [6] or as a massive Yang-Mills field [7]. In both cases the soliton was shown to be *unstable* and consequently one can not describe the excited states, in particular the breathing mode. In a recent paper [8] it was proven that the soliton is stable with respect to the breathing fluctuations if the ρ -meson is introduced in the chiral Lagrangian following the standard prescription of nonlinear chiral symmetry for massive particles [9]. In Ref. [8] however, the ρ -meson is described in terms of antisymmetric tensor field as proposed by the authors of Refs. [10, 11]. It has been shown in Ref. [12] that this description is canonically equivalent to a vector field formulation provided the Skyrme term [13] is added to the latter in order to keep the corresponding Hamiltonian bounded. In this work we make use of this last formulation to introduce the ρ -meson. It yields:

$$\mathcal{L}_{\pi\rho} = \frac{M_\rho^2}{2} \text{Tr}(V_\mu V^\mu) - \frac{1}{4} \text{Tr} \left\{ \left(\nabla_\mu V_\nu - \nabla_\nu V_\mu + i \frac{g_V}{\sqrt{2}} [u_\mu, u_\nu] \right)^2 \right\} \quad (3)$$

where V_μ is the ρ -meson field and

$$\nabla_\mu = \partial_\mu + [\Gamma_\mu, \cdot], \quad \Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u], \quad u = U^{\frac{1}{2}}, \quad u_\mu = i u^\dagger \partial_\mu U u^\dagger . \quad (4)$$

The two constants appearing in Eq. (3) are the ρ -meson mass M_ρ and the dimensionless parameter g_V which can be related to the $\rho \rightarrow \pi\pi$ decay width.

The third term in Eq. (1) corresponds to ω -meson exchange [4]. It reads:

$$\mathcal{L}_{\pi\omega} = -\frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \beta_\omega\omega_\mu B^\mu \quad (5)$$

where ω_μ is the omega field and B^μ the baryon current [13, 14]

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\text{Tr}\left\{(\partial_\nu U)U^\dagger(\partial_\alpha U)U^\dagger(\partial_\beta U)U^\dagger\right\}. \quad (6)$$

The two new constants appearing in Eq. (5) are the ω -meson mass m_ω and the parameter β_ω which can be related to the $\omega \rightarrow \pi\gamma$ width [15]. Finally, the last term in the Lagrangian (1) implements a small explicit breaking of chiral symmetry, m_π being the pion mass.

2. In order to work with true degrees of freedom and eliminate the constrained fields V_0 and ω_0 (the corresponding canonical momenta conjugate are equal to zero), it is suitable to consider the Hamiltonian density. Let $\vec{\Phi}, P^i$ and p^i being the canonical momenta conjugate to the pion field $\vec{\pi}$, the ρ field and the ω field respectively. We make the hedgehog ansatz for the pion and the most general spherical ansatz for the ρ and the ω fields:

$$\begin{aligned} \vec{\pi} &= f_\pi F \hat{\mathbf{r}}, \quad V_i = f_\pi \{v_1(\tau_i - (\vec{\tau}\cdot\hat{\mathbf{r}})\hat{r}_i) + v_2(\vec{\tau}\cdot\hat{\mathbf{r}})\hat{r}_i - v_3(\vec{\tau}\times\hat{\mathbf{r}})_i\}, \quad \omega_i = f_\pi\omega \hat{r}_i, \\ \vec{\Phi} &= \frac{f_\pi}{r}\phi \hat{\mathbf{r}}, \quad P_i = \frac{f_\pi}{r}\{\gamma_1(\tau_i - (\vec{\tau}\cdot\hat{\mathbf{r}})\hat{r}_i) + \gamma_2(\vec{\tau}\cdot\hat{\mathbf{r}})\hat{r}_i - \gamma_3(\vec{\tau}\times\hat{\mathbf{r}})_i\}, \quad p_i = \frac{f_\pi}{r}p \hat{r}_i. \end{aligned} \quad (7)$$

The energy functional expressed in terms of the time-dependent radial functions $F, \phi, v_i, \gamma_i, \omega, p$ reads:

$$\begin{aligned} E = & 4\pi f_\pi^2 \int_0^\infty dr \left\{ \frac{1}{2}\gamma_1^2 + \frac{1}{4}\gamma_2^2 + \frac{1}{2}\gamma_3^2 + M_\rho^2 r^2 (2v_1^2 + v_2^2) + \frac{1}{2}m_\omega^2 r^2 \omega^2 + \right. \\ & \frac{1}{2}(\phi + 2\bar{g}\frac{s}{r}\gamma_3 - \bar{\beta}\frac{s^2}{r}\omega)^2 + 2((rv_1)' - v_2c)^2 + \frac{1}{4M_\rho^2 r^2}(2\gamma_1c - (\gamma_2r)')^2 + \\ & \frac{1}{2}r^2 F'^2 + s^2 + 2M_\rho^2 r^2 v_3^2 + \frac{1}{2}p^2 + m_\pi^2 r^2 (1-c) + \\ & \left. (2v_3c + \bar{g}\frac{s^2}{r})^2 + 2(\bar{g}sF' + (rv_3)')^2 + \frac{1}{2m_\omega^2 r^2}(\bar{\beta}s^2F' + (rp)')^2 \right\} \end{aligned} \quad (8)$$

with $\bar{g} = \sqrt{2}g_V/f_\pi$, $\bar{\beta} = \beta_\omega/2\pi^2 f_\pi$, $s = \sin F$ and $c = \cos F$. Primes indicate radial differentiations.

By solving the static Hamilton equations, i.e.,

$$\dot{F} = \dot{\phi} = \dot{v}_i = \dot{\gamma}_i = \dot{\omega} = \dot{p} = 0$$

where dots indicate partial time differentiations, we find that the only nonsingular solution which extremizes E is the one which corresponds to

$$\phi(r) = v_1 = v_2 = \gamma_1 = \gamma_2 = \gamma_3 = \omega = 0. \quad (9)$$

The set of the three other static Hamilton equations corresponding to F, v_3 and p reads:

$$\begin{aligned} r(rF)'' + 4\bar{g}\frac{s}{r}(M_\rho^2 r^2 + s^2)v_3 + 2sc(4v_3^2 - 1) + \bar{\beta}\frac{s^2}{r}p - m_\pi^2 r^2 s &= 0, \\ r(rv_3)'' - (2c^2 + M_\rho^2 r^2)v_3 - \bar{g}(\frac{cs^2}{r} + rc'') &= 0, \\ r^2 p'' - (2 + m_\omega^2 r^2)p + r^3 \bar{\beta}(\frac{s^2}{r^2}F')' &= 0. \end{aligned} \quad (10)$$

We solve numerically the equations (10) with the boundary conditions $F(0) = \pi$, $F(\infty) = 0$ in order to find a winding number one solution. In Fig. 1 we plot that solution for the following set of physical parameters (fixed by fitting to the low energy meson observables) :

$$\begin{aligned} f_\pi = 93, \quad M_\rho = 769, \quad m_\omega = 782, \quad m_\pi = 139.5 \quad (\text{MeV}) \\ \beta_\omega = 9.3, g_V = 0.09. \end{aligned} \quad (11)$$

The classical soliton mass we find is $M = 1515$ MeV, and is lower than the mass which corresponds to the local approximation (1714 MeV) that one finds if the masses M_ρ and m_ω and the coupling constant β_ω are increased to infinity, keeping g_V and the ratio β_ω/m_ω finites. We also plot in Fig. 1 the local approximation's chiral function for reference.

3. In order to investigate the soliton breathing mode within the model (1) we consider the linearized time-dependent Hamilton equations in presence of an external infinitesimal monopole field with a frequency Ω . This can be done by adding the following perturbation term [1, 5]

$$\delta E_{\text{int}} = \epsilon f_\pi^3 \langle r^2 \rangle \sin(\Omega t) \exp(\eta t) \quad (12)$$

to the energy functional (8). In Eq. (12), $\langle r^2 \rangle$ is the isoscalar mean square radius [16] and η a vanishingly small positive number.

Because the term (12) is weak it introduces only small changes of the classical Hamilton solution. We thus can use the linear approximation and look for a solution in the form

$$\begin{aligned} F(r, t) &= F(r) + \epsilon [\delta F(r) e^{i(\Omega - i\eta)t} - c.c.] / 2i, \quad \phi(r, t) = \phi(r) + \epsilon [\delta \phi(r) e^{i(\Omega - i\eta)t} - c.c.] / 2i, \\ \omega(r, t) &= \omega(r) + \epsilon [\delta \omega(r) e^{i(\Omega - i\eta)t} - c.c.] / 2i, \quad p(r, t) = p(r) + \epsilon [\delta p(r) e^{i(\Omega - i\eta)t} - c.c.] / 2i, \\ v_i(r, t) &= v_i(r) + \epsilon [\delta v_i(r) e^{i(\Omega - i\eta)t} - c.c.] / 2i, \quad \gamma_i(r, t) = \gamma_i(r) + \epsilon [\delta \gamma_i(r) e^{i(\Omega - i\eta)t} - c.c.] / 2i, \end{aligned}$$

with the boundary conditions that all the fluctuations and their time derivatives are equal to zero at $t = -\infty$. We are not going to write all the ten linearized equations but it is straightforward to show from Eq. (8) that the equations corresponding to the fluctuations $\delta\phi, \delta\omega, \delta\gamma_i, \delta v_1$ and δv_2 decouple from those which correspond to $\delta F, \delta v_3$ and δp , and consequently do not contribute to the breathing

mode. The equations corresponding to the breathing fluctuations δF , δv_3 and δp read :

$$\left[\begin{array}{ccc} (\Omega - i\eta)^2 & - & \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{pmatrix} \end{array} \right] \begin{pmatrix} \delta F \\ \delta\psi \\ \delta\xi \end{pmatrix} = \frac{f_\pi s^2}{\pi^2 r} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (13)$$

where $\delta\psi$ and $\delta\xi$ are combinations of δv_3 and δp , namely

$$\delta\psi = \delta v_3 + \bar{g} \frac{s}{r} \delta F, \quad \delta\xi = \delta p + \bar{\beta} \frac{s^2}{r} \delta F.$$

The matrix elements A_{ij} of the operator \mathcal{A} appearing in Eq. (13) are themselves operators and read as:

$$\begin{aligned} A_{11} &= -\frac{1}{r} \frac{d^2}{dr^2} r + \frac{4}{r^2} \left[(2v_3^2 - \frac{1}{2})(s^2 - c^2) + \frac{\bar{g}}{r} s^2 (\frac{\bar{g}}{r} s^2 + v_3 c) + \frac{\bar{g}}{r} M_\rho^2 r^2 (\frac{\bar{g}}{r} s^2 - v_3 c) \right. \\ &\quad \left. - \frac{\bar{\beta}}{4r} s(2cp - \frac{\bar{\beta}}{r} s^3) + \frac{1}{4} m_\pi^2 r^2 c \right] \\ A_{12} &= -\frac{4s}{r^2} \left[\frac{\bar{g}}{r} (s^2 + M_\rho^2 r^2) + 4v_3 c \right] \\ A_{13} &= -\bar{\beta} \frac{s^2}{r^3} \\ A_{21} &= -\frac{s}{r^2} \left[\frac{\bar{g}}{r} (s^2 + M_\rho^2 r^2) + 4v_3 c \right] \\ A_{22} &= -\frac{1}{r} \frac{d^2}{dr^2} r + \frac{1}{r^2} (2c^2 + M_\rho^2 r^2) \\ A_{31} &= -m_\omega^2 \bar{\beta} \frac{s^2}{r} \\ A_{33} &= -\frac{d^2}{dr^2} + \frac{1}{r^2} (m_\omega^2 r^2 + 2) . \end{aligned} \quad (14)$$

We recall that s means $\sin(F)$, c stands for $\cos(F)$ and F is the static solution for the pion field.

The monopole response function is determined from the evolution of the isoscalar mean square radius of the soliton with respect to the frequency Ω . The mean square radius is given by [16]

$$\langle r^2 \rangle(t) = \int d^3r B^0(r, t) r^2 = -\frac{2}{\pi} \int_0^\infty dr r^2 \sin^2(F) F'$$

where $B^0(r, t)$ is the time-component of the baryon current (6). Up to first order in ϵ , $\langle r^2 \rangle$ reads

$$\langle r^2 \rangle(t) = \langle r^2 \rangle_0 + \frac{\epsilon}{2i} [f(\Omega) e^{i(\Omega - i\eta)t} - c.c]$$

where f is the linear response function

$$f(\Omega) = \frac{4}{\pi} \int_0^\infty dr r \sin^2(F) \delta F(r). \quad (15)$$

The spectral representation of the response function f can be extracted by using equations (13). It reads

$$f(\Omega) = \frac{1}{\pi} \sum_n \frac{|\langle \chi | \chi_n \rangle|^2}{(\Omega - i\eta)^2 - \lambda_n^2}, \quad (16)$$

where the limit $\eta \rightarrow 0^+$ is, as usual, implicit, and corresponds to the boundary conditions specified above. In Eq.(16) the state χ is defined by

$$\langle \chi | r \rangle = \frac{2}{\pi f_\pi} \frac{s^2}{r} (1, 0, 0) \quad (17)$$

and the χ_n are the eigenstates of the operator \mathcal{A} (see Eqs. (13),(14)), with the eigenvalues λ_n^2 , normalized according to

$$\langle \chi_n | \chi_m \rangle = \int_0^\infty f_\pi^3 r^2 dr \chi_n^+(r) \chi_m(r) = \delta_{nm} \quad .$$

Since the distribution of collective strength is directly related to the imaginary part of the linear response function, we should consider this quantity. We display in Fig. 2 the imaginary part of the function (16) with respect to the frequency Ω for the parameters given in Eq. (11). We find that it exhibits a pronounced peak at 400 MeV which we identify with the excitation energy of the Roper resonance. In Ref. [1] we have found nearly the same value within the local approximation to the Lagrangian (1) except for the detail that the dimensionless Skyrme term parameter e [13] taken in [1] is equal to 7.1 [17, 18], while the value of e which corresponds to the parameters (11) we use in this work is $e = 1/2g_V = 5.57$. We show in Fig. 2 the response function which corresponds to the local approximation [1] taking $e = 5.57$. One sees from this figure that there is an improvement over the local case, although a slight one (~ 40 MeV) but in the good direction. A more realistic improvement consists of extending the model (1) to include other mesons such as the vector meson- A_1 and the scalar meson- ϵ despite of the increasing complexity implied.

4. To summarize, we have investigated the soliton breathing mode within an effective Lagrangian with finite mass mesons (the ρ - and the ω -mesons) in the framework of linear response theory. Our results for the location of the Roper resonance show an improvement over the local approximation where these two mesons are taken to be infinitely heavy.

It is quite well established (see for example [15]) that for the description of static nucleon properties, finite mass meson Lagrangians are more accurate than their local residues, such as the Skyrme model. As this work suggests, this feature is also true for the description of dynamical observables such as the breathing mode.

Finally, the difficulty to find the $P11$ resonance $N(1440)$ in analysis based on phase shifts method [19] and the encouraging results obtained in this work and in Refs. [1, 5] lead us to think that the linear response theory is the most adapted method to describe the Roper resonance within effective Lagrangians.

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Figure captions

FIG. 1. The solution of Eq. (10) minimizing the energy functional (8). The full line displays the static function $F(r)$ in this model, while the dotted one corresponds to the local approximation of (1). In the dashed and dashed-dotted lines we display the ρ and the ω degrees of freedom respectively. Namely $v_3(r)$ and $p(r)$.

FIG. 2. Imaginary part of the response function f (fm^2) versus the energy Ω in MeV. The full line corresponds to the Lagrangian (1) with the parameters given in Eq. (11) while the dotted line corresponds to the local approximation of (1) with $f_\pi = 93$ MeV, $\beta_\omega = 9.3$ and $e = 1/2g_V = 5.57$ (c.f. Ref. [1]).

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